

# ECE 443/643 Homework 1

August 29, 2011

*General hint: remember Euler's formula  $e^{j\theta} = \cos \theta + j \sin \theta$ .*

1. Let  $p(t) = -t\Pi(t - \frac{1}{2}) = t(u(t) - u(t - 1))$ .
  - (a) Sketch  $p(t)$ ,  $f(t) = -p(-t)$ ,  $g(t) = p(\frac{1}{2}t)$ ,  $h(t) = p(2t)$ , and  $m(t) = 3p(2(t + 1))$ .
  - (b) Find the energy of each signal in part (a). *Hint: it may be faster to determine how various transformations of a signal change the energy.*
  - (c) Let  $s(t) = p(t) - p(-t)$ ,  $x(t) = \sum_{n=-N}^N s(t - 2n)$ , and  $y(t) = \sum_{n=-N}^N s(t - 3n)$ . Sketch the three signals for  $N = 2$ .
  - (d) Find the energy of each of the signals in part (c) for arbitrary  $N$ .
2. Find the energy and power of the given signals, and indicate if they are energy signals, power signals, or neither.
  - (a)  $j2e^{-3|t|}$
  - (b)  $j2e^{(-4+j^2)t}u(-t)$
  - (c)  $3e^{-|t|}\cos(2t)$
  - (d)  $j5e^{-j3t}$
3. Evaluate the following expressions:
  - (a)  $\int_{-\infty}^{\infty} e^{-t^3}\delta(t - 3)dt$
  - (b)  $\int_{-\infty}^{\infty} \cos(5t^2)\delta(2t - 3)dt$
  - (c)  $\int_{-\infty}^{5/3} \cos(5t^2)\delta(2t - 3)dt$
  - (d)  $\int_t^{\infty} \tau^2\delta(\tau - 1)d\tau$
4. Let  $p(t) = u(t) - u(t - 1)$ ,  $\phi(t) = tp(t)$ ,  $x(t) = \sin(\frac{\pi}{2}t)p(t)$ , and  $\hat{x}(t) = c\phi(t)$ . Find  $c \in \mathbb{R}$  to minimize

$$\mathcal{E} = \|x - \hat{x}\|^2 = \int_{-\infty}^{\infty} |x(t) - \hat{x}(t)|^2 dt.$$

*Hint: think of  $\mathcal{E}(c)$  as a real-valued function of a real variable.*

5. Let  $\langle x, y \rangle = \frac{1}{10} \int_0^{10} x(t)y^*(t)dt$ , and let  $\phi_n(t) = e^{2\pi j n \frac{t}{10}}$ .
- (a) Compute and simplify  $\langle \phi_n, \phi_m \rangle$  for all integers  $n$  and  $m$ .
  - (b) Let  $x(t) = \cos(7\pi t)$  and find  $X_n = \langle x, \phi_n \rangle$  for all integer  $n$ .
  - (c) Find  $\sum_{n=-\infty}^{\infty} |X_n|^2$ .
  - (d) Let  $y(t) = 5 \sin(9\pi t - \frac{\pi}{4})$  and find  $Y_n = \langle y, \phi_n \rangle$ .
  - (e) Find  $\sum_{n=-\infty}^{\infty} |Y_n|^2$ .
  - (f) Find  $\langle x, y \rangle$ ,  $\langle x, x \rangle$ , and  $\langle y, y \rangle$ .